Mathematica 11.3 Integration Test Results

Test results for the 23 problems in "4.6.1.4 (d cot)^n (a+b csc)^m.m"

Problem 1: Result more than twice size of optimal antiderivative.

$$\int \frac{\mathrm{Tan}[x]^4}{a + a\,\mathrm{Csc}[x]}\,dx$$

Optimal (type 3, 55 leaves, 5 steps):

$$\frac{x}{a} - \frac{(15 - 8\,\mathrm{Csc}[x])\,\mathrm{Tan}[x]}{15\,a} + \frac{(5 - 4\,\mathrm{Csc}[x])\,\mathrm{Tan}[x]^3}{15\,a} - \frac{(1 - \mathrm{Csc}[x])\,\mathrm{Tan}[x]^5}{5\,a}$$

Result (type 3, 111 leaves):

$$\left(200 + 6\,(-89 + 120\,x)\,\mathrm{Cos}[x] + 128\,\mathrm{Cos}[2\,x] - 178\,\mathrm{Cos}[3\,x] + 240\,x\,\mathrm{Cos}[3\,x] + 184\,\mathrm{Cos}[4\,x] - 64\,\mathrm{Sin}[x] - 178\,\mathrm{Sin}[2\,x] + 240\,x\,\mathrm{Sin}[2\,x] - 128\,\mathrm{Sin}[3\,x] - 89\,\mathrm{Sin}[4\,x] + 120\,x\,\mathrm{Sin}[4\,x]\right) \Big/ \left(960\,a\left(\mathrm{Cos}\left[\frac{x}{2}\right] - \mathrm{Sin}\left[\frac{x}{2}\right]\right)^3 \left(\mathrm{Cos}\left[\frac{x}{2}\right] + \mathrm{Sin}\left[\frac{x}{2}\right]\right)^5\right)$$

Problem 5: Result more than twice size of optimal antiderivative.

$$\int \frac{\mathrm{Cot}[x]}{a + a\,\mathrm{Csc}[x]}\,dx$$

Optimal (type 3, 9 leaves, 2 steps):

$$\frac{\mathrm{Log}[1 + \mathrm{Sin}[x]]}{a}$$

Result (type 3, 19 leaves):

$$\frac{2\,\mathrm{Log}\left[\mathrm{Cos}\left[\frac{x}{2}\right] + \mathrm{Sin}\left[\frac{x}{2}\right]\right]}{a}$$

Problem 7: Result more than twice size of optimal antiderivative.

$$\int \frac{\mathrm{Cot}[x]^3}{a + a\,\mathrm{Csc}[x]}\,dx$$

Optimal (type 3, 16 leaves, 3 steps):

$$-\frac{\mathrm{Csc}[x]}{a} - \frac{\mathrm{Log}[\mathrm{Sin}[x]]}{a}$$

Result (type 3, 35 leaves):

$$-\frac{\text{Cot}\left[\frac{x}{2}\right]}{2\,a}-\frac{\text{Log}[\text{Sin}[x]]}{a}-\frac{\text{Tan}\left[\frac{x}{2}\right]}{2\,a}$$

Problem 8: Result more than twice size of optimal antiderivative.

$$\int \frac{\text{Cot}[x]^4}{a+a\,\text{Csc}[x]}\,dx$$

Optimal (type 3, 31 leaves, 4 steps):

$$\frac{x}{a}+\frac{\text{ArcTanh}[\text{Cos}[x]]}{2\,a}+\frac{\text{Cot}[x]\,(2-\text{Csc}[x])}{2\,a}$$

Result (type 3, 90 leaves):

$$\frac{x}{a}+\frac{\text{Cot}\left[\frac{x}{2}\right]}{2\,a}-\frac{\text{Csc}\left[\frac{x}{2}\right]^2}{8\,a}+\frac{\text{Log}\left[\text{Cos}\left[\frac{x}{2}\right]\right]}{2\,a}-\frac{\text{Log}\left[\text{Sin}\left[\frac{x}{2}\right]\right]}{2\,a}+\frac{\text{Sec}\left[\frac{x}{2}\right]^2}{8\,a}-\frac{\text{Tan}\left[\frac{x}{2}\right]}{2\,a}$$

Problem 9: Result more than twice size of optimal antiderivative.

$$\int \frac{\text{Cot}[x]^5}{a+a\,\text{Csc}[x]}\,dx$$

Optimal (type 3, 36 leaves, 3 steps):

$$\frac{\text{Csc}[x]}{a}+\frac{\text{Csc}[x]^2}{2\,a}-\frac{\text{Csc}[x]^3}{3\,a}+\frac{\text{Log}[\text{Sin}[x]]}{a}$$

Result (type 3, 106 leaves):

$$\frac{5\,\text{Cot}\left[\frac{x}{2}\right]}{12\,a}+\frac{\text{Csc}\left[\frac{x}{2}\right]^2}{8\,a}-\frac{\text{Cot}\left[\frac{x}{2}\right]\text{Csc}\left[\frac{x}{2}\right]^2}{24\,a}+\frac{\text{Log}[\text{Sin}[x]]}{a}+\frac{\text{Sec}\left[\frac{x}{2}\right]^2}{8\,a}+\frac{5\,\text{Tan}\left[\frac{x}{2}\right]}{12\,a}-\frac{\text{Sec}\left[\frac{x}{2}\right]^2\text{Tan}\left[\frac{x}{2}\right]}{24\,a}$$

Problem 10: Result more than twice size of optimal antiderivative.

$$\int \frac{\text{Cot}[x]^6}{a+a\,\text{Csc}[x]}\,dx$$

Optimal (type 3, 49 leaves, 5 steps):

$$-\frac{x}{a}-\frac{3\,\text{ArcTanh}[\text{Cos}[x]]}{8\,a}+\frac{\text{Cot}[x]^3\,(4-3\,\text{Csc}[x])}{12\,a}-\frac{\text{Cot}[x]\,(8-3\,\text{Csc}[x])}{8\,a}$$

Result (type 3, 163 leaves):

$$-\frac{x}{a}-\frac{2\,\text{Cot}\left[\frac{x}{2}\right]}{3\,a}+\frac{5\,\text{Csc}\left[\frac{x}{2}\right]^2}{32\,a}+\frac{\text{Cot}\left[\frac{x}{2}\right]\text{Csc}\left[\frac{x}{2}\right]^2}{24\,a}-\frac{\text{Csc}\left[\frac{x}{2}\right]^4}{64\,a}-\frac{3\,\text{Log}\left[\text{Cos}\left[\frac{x}{2}\right]\right]}{8\,a}+$$
$$\frac{3\,\text{Log}\left[\text{Sin}\left[\frac{x}{2}\right]\right]}{8\,a}-\frac{5\,\text{Sec}\left[\frac{x}{2}\right]^2}{32\,a}+\frac{\text{Sec}\left[\frac{x}{2}\right]^4}{64\,a}+\frac{2\,\text{Tan}\left[\frac{x}{2}\right]}{3\,a}-\frac{\text{Sec}\left[\frac{x}{2}\right]^2\text{Tan}\left[\frac{x}{2}\right]}{24\,a}$$

Problem 11: Result more than twice size of optimal antiderivative.

$$\int \frac{\text{Cot}[x]^7}{a + a\,\text{Csc}[x]}\,dx$$

Optimal (type 3, 58 leaves, 3 steps):

$$-\frac{\text{Csc}[x]}{a} - \frac{\text{Csc}[x]^2}{a} + \frac{2\,\text{Csc}[x]^3}{3\,a} + \frac{\text{Csc}[x]^4}{4\,a} - \frac{\text{Csc}[x]^5}{5\,a} - \frac{\text{Log}[\text{Sin}[x]]}{a}$$

Result (type 3, 179 leaves):

$$-\frac{89\,\text{Cot}\left[\frac{x}{2}\right]}{240\,a} - \frac{7\,\text{Csc}\left[\frac{x}{2}\right]^2}{32\,a} + \frac{31\,\text{Cot}\left[\frac{x}{2}\right]\,\text{Csc}\left[\frac{x}{2}\right]^2}{480\,a} + \frac{\text{Csc}\left[\frac{x}{2}\right]^4}{64\,a} - \frac{\text{Cot}\left[\frac{x}{2}\right]\,\text{Csc}\left[\frac{x}{2}\right]^4}{160\,a} - \frac{\text{Log}[\text{Sin}[x]]}{a} - \frac{7\,\text{Sec}\left[\frac{x}{2}\right]^2}{32\,a} + \frac{\text{Sec}\left[\frac{x}{2}\right]^4}{64\,a} - \frac{89\,\text{Tan}\left[\frac{x}{2}\right]}{240\,a} + \frac{31\,\text{Sec}\left[\frac{x}{2}\right]^2\,\text{Tan}\left[\frac{x}{2}\right]}{480\,a} - \frac{\text{Sec}\left[\frac{x}{2}\right]^4\,\text{Tan}\left[\frac{x}{2}\right]}{160\,a}$$

Problem 12: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\text{Tan}[x]^5}{a + b\,\text{Csc}[x]}\,dx$$

Optimal (type 3, 178 leaves, 3 steps):

$$\frac{1}{16\,(a+b)\,(1-\text{Csc}[x])^2} + \frac{5\,a+7\,b}{16\,(a+b)^2\,(1-\text{Csc}[x])} + \frac{1}{16\,(a-b)\,(1+\text{Csc}[x])^2} + \frac{5\,a-7\,b}{16\,(a-b)^2\,(1+\text{Csc}[x])} - \frac{(8\,a^2+21\,a\,b+15\,b^2)\,\text{Log}[1-\text{Csc}[x]]}{16\,(a+b)^3} - \frac{(8\,a^2-21\,a\,b+15\,b^2)\,\text{Log}[1+\text{Csc}[x]]}{16\,(a-b)^3} + \frac{b^6\,\text{Log}[a+b\,\text{Csc}[x]]}{a\,(a^2-b^2)^3} - \frac{\text{Log}[\text{Sin}[x]]}{a}$$

Result (type 3, 301 leaves):

$$\frac{1}{16\,(a+b\,\text{Csc}[x])}\text{Csc}[x]\left(-\frac{32\,i\,(a^5-3\,a^3\,b^2+3\,a\,b^4)\,x}{(a-b)^3\,(a+b)^3} - \frac{2\,i\,(8\,a^2-21\,a\,b+15\,b^2)\,\text{ArcTan}[\text{Cot}[x]]}{(a-b)^3} - \frac{2\,i\,(8\,a^2+21\,a\,b+15\,b^2)\,\text{ArcTan}[\text{Cot}[x]]}{(a+b)^3} + \frac{(8\,a^2-21\,a\,b+15\,b^2)\,\text{Log}\left[\left(\text{Cos}\left[\frac{x}{2}\right]+\text{Sin}\left[\frac{x}{2}\right]\right)^2\right]}{(-a+b)^3} - \frac{(8\,a^2+21\,a\,b+15\,b^2)\,\text{Log}[1-\text{Sin}[x]]}{(a+b)^3} + \frac{16\,b^6\,\text{Log}[b+a\,\text{Sin}[x]]}{a\,(a^2-b^2)^3} + \frac{1}{(a+b)\,\left(\text{Cos}\left[\frac{x}{2}\right]-\text{Sin}\left[\frac{x}{2}\right]\right)^4} + \frac{1}{(a-b)\,\left(\text{Cos}\left[\frac{x}{2}\right]+\text{Sin}\left[\frac{x}{2}\right]\right)^4} + \frac{-7\,a+9\,b}{(a-b)^2\,\left(\text{Cos}\left[\frac{x}{2}\right]+\text{Sin}\left[\frac{x}{2}\right]\right)^2} + \frac{7\,a+9\,b}{(a+b)^2\,(-1+\text{Sin}[x])}\right)(b+a\,\text{Sin}[x])$$

Problem 17: Result more than twice size of optimal antiderivative.

$$\int \frac{\text{Cot}[x]^5}{a+b\,\text{Csc}[x]}\,dx$$

Optimal (type 3, 72 leaves, 3 steps):

$$-\frac{\left(a^2-2\,b^2\right)\,\text{Csc}[x]}{b^3}+\frac{a\,\text{Csc}[x]^2}{2\,b^2}-\frac{\text{Csc}[x]^3}{3\,b}+\frac{\left(a^2-b^2\right)^2\,\text{Log}[a+b\,\text{Csc}[x]]}{a\,b^4}+\frac{\text{Log}[\text{Sin}[x]]}{a}$$

Result (type 3, 179 leaves):

$$\frac{1}{48\,a\,b^4}\left(\left(-24\,a^3\,b+44\,a\,b^3\right)\,\text{Cot}\left[\frac{x}{2}\right]+6\,a^2\,b^2\,\text{Csc}\left[\frac{x}{2}\right]^2-48\,a^4\,\text{Log}[\text{Sin}[x]]+96\,a^2\,b^2\,\text{Log}[\text{Sin}[x]]+48\,a^4\,\text{Log}[b+a\,\text{Sin}[x]]-96\,a^2\,b^2\,\text{Log}[b+a\,\text{Sin}[x]]+48\,b^4\,\text{Log}[b+a\,\text{Sin}[x]]+6\,a^2\,b^2\,\text{Sec}\left[\frac{x}{2}\right]^2-16\,a\,b^3\,\text{Csc}[x]^3\,\text{Sin}\left[\frac{x}{2}\right]^4-a\,b^3\,\text{Csc}\left[\frac{x}{2}\right]^4\,\text{Sin}[x]-24\,a^3\,b\,\text{Tan}\left[\frac{x}{2}\right]+44\,a\,b^3\,\text{Tan}\left[\frac{x}{2}\right]\right)$$

Problem 18: Result more than twice size of optimal antiderivative.

$$\int \frac{\text{Cot}[x]^7}{a+b\,\text{Csc}[x]}\,dx$$

Optimal (type 3, 122 leaves, 3 steps):

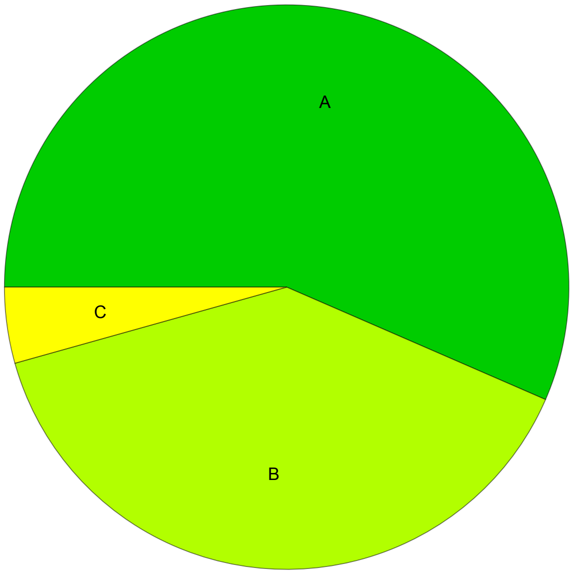
$$-\frac{\left(a^4-3\,a^2\,b^2+3\,b^4\right)\,\text{Csc}[x]}{b^5}+\frac{a\,\left(a^2-3\,b^2\right)\,\text{Csc}[x]^2}{2\,b^4}-\frac{\left(a^2-3\,b^2\right)\,\text{Csc}[x]^3}{3\,b^3}+\frac{a\,\text{Csc}[x]^4}{4\,b^2}-\frac{\text{Csc}[x]^5}{5\,b}+\frac{\left(a^2-b^2\right)^3\,\text{Log}[a+b\,\text{Csc}[x]]}{a\,b^6}-\frac{\text{Log}[\text{Sin}[x]]}{a}$$

Result (type 3, 343 leaves):

$$-\frac{1}{960\,a\,b^6}\left(4\,a\,b\,\left(120\,a^4-340\,a^2\,b^2+309\,b^4\right)\,\text{Cot}\left[\frac{x}{2}\right]-30\,a^2\,b^2\,\left(4\,a^2-11\,b^2\right)\,\text{Csc}\left[\frac{x}{2}\right]^2+960\,a^6\,\text{Log}[\text{Sin}[x]]-2880\,a^4\,b^2\,\text{Log}[\text{Sin}[x]]+2880\,a^2\,b^4\,\text{Log}[\text{Sin}[x]]-960\,a^6\,\text{Log}[b+a\,\text{Sin}[x]]+2880\,a^4\,b^2\,\text{Log}[b+a\,\text{Sin}[x]]-2880\,a^2\,b^4\,\text{Log}[b+a\,\text{Sin}[x]]+960\,b^6\,\text{Log}[b+a\,\text{Sin}[x]]-120\,a^4\,b^2\,\text{Sec}\left[\frac{x}{2}\right]^2+330\,a^2\,b^4\,\text{Sec}\left[\frac{x}{2}\right]^2-15\,a^2\,b^4\,\text{Sec}\left[\frac{x}{2}\right]^4+320\,a^3\,b^3\,\text{Csc}[x]^3\,\text{Sin}\left[\frac{x}{2}\right]^4-816\,a\,b^5\,\text{Csc}[x]^3\,\text{Sin}\left[\frac{x}{2}\right]^4+3\,a\,b^5\,\text{Csc}\left[\frac{x}{2}\right]^6\,\text{Sin}[x]+a\,b^3\,\text{Csc}\left[\frac{x}{2}\right]^4\,\left(-15\,a\,b+20\,a^2\,\text{Sin}[x]-51\,b^2\,\text{Sin}[x]\right)+480\,a^5\,b\,\text{Tan}\left[\frac{x}{2}\right]-1360\,a^3\,b^3\,\text{Tan}\left[\frac{x}{2}\right]+1236\,a\,b^5\,\text{Tan}\left[\frac{x}{2}\right]+6\,a\,b^5\,\text{Sec}\left[\frac{x}{2}\right]^4\,\text{Tan}\left[\frac{x}{2}\right]\right)$$

Summary of Integration Test Results

23 integration problems



A - 13 optimal antiderivatives

B - 9 more than twice size of optimal antiderivatives

C - 1 unnecessarily complex antiderivatives

D - 0 unable to integrate problems

E - 0 integration timeouts